A Corpus-Based Model of Voice Leading in Tonal Music

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Keywords: hidden Markov model, music theory pedagogy, tonal harmony, part writing, harmonic function, scale-degree function

Abstract

This study proposes an empirically derived model of tonal voice leading that can be used to teach part writing in undergraduate harmony courses. Machine-learning techniques were used to divide scale degrees into classes based on their voice-leading tendencies. The resulting theory can therefore be seen as an extension of Harrison's (1994) models of voice leading based on harmonic function, Quinn's (2005) "Harmonic Function without Primary Triads", and Shaffer's (2014) scale-degree categories in *Open Music Theory, Version 1*. The dataset used in this study was derived from MIDI files of four-voice chorale-style hymns selected from *The Cyber Hymnal*. An algorithm for determining harmonic function created a list of scale degree numbers with their harmonic functional context for each voice. The machine learning produced a hidden Markov model with six states that can be interpreted as scale-degree functions. Division of the model into 21 classes allowed for interpretation of the voice-leading tendencies of each of the original six classes in all harmonic contexts. Using these conclusions, this study proposes a system of analysis and a simple set of voice-leading procedures that undergraduate music majors can learn in order to master the technique of tonal part writing and analysis.

Introduction

How does a student just learning how to part write approach the task differently from an expert? The expert's experience provides them a repertoire of high-level voice-leading schemata that they can draw upon in any situation. An inexperienced student, however, has no such prelearned voice-leading patterns. Instead, a student will likely make a list of the notes in each new chord, try one of these notes in the first voice, determine the consequences of this choice in the remaining voices, and, if this fails, alter their choice and try again. This approach does not ensure that all viable solutions are examined and evaluated.

If a music theory teacher wants to help a student toward mastery of part writing, they should try to build upon the student's current approach, and progress from there toward the goal of the expert's method. This study introduces a part-writing methodology that is based on the inexperienced student's voice-by-voice approach to part writing, giving specific guidance on the direction and distance of resolution of each note in a chord. The voice-leading procedures that I propose are derived from an empirical study of the voice-leading practices of hymn composers.

Methods

The corpus used to generate this voice-leading model is the collection of church hymns at *The Cyber Hymnal* (http://hymntime.com). From the hymn database, I hand-selected 1582 MIDI files that are both major and minor, and in four-voice chorale-style (homorhythmic). I needed the pertinent information from these MIDI files in a form that the computer could process to generate an analysis of their voice leading. To do this, I applied a computer algorithm that turned each MIDI file into a list of the notes in the MIDI file, each represented as the harmonic function of the surrounding chord and its scale-degree number followed by a semicolon.

An example of this data format appears in Figure 1. Although the chords are lined up vertically in this example, they don't need to be, since the computer is reading one note at a time in left-to-right order. The harmonic analysis information that the computer needs is built into the pitch's symbolic representation. The computer can therefore look at the succession of scale degrees to see which of these commonly follow each one. I then used this training data to generate a hidden Markov model of the hidden states—or scale-degree categories—that can be used to predict each new scale degree in the dataset.

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#americus.mid
T5; T5; S6; T5; T3; T3; S4; T3; T3; D4; T5; S6; T5; D4; T3; D2; D2; D5; T6; D7; S6; D5; D5; T6; D7; T1; T5; S6; T7; S6; S2; T3; D4; T5; T5; D5; D4; T3; T1; S1; T1; S1; S2; T3; T3; T5; S4; D3; D2; T1;
T1; T1; S1; T1; T1; T1; S1; T1; T1; D7; T1; S1; T1; D7; T1; D7; D7; D2; T3; D2; S1; D7; D7; T1; D2; T3; T1; S1; T1; S1; S1; T1; D7; T1; T1; D7; D2; T1; T5; S5; T7; S6; S6; T5; T1; T1; S1; D1; D7; T1;
T3; T3; S4; T3; T5; T5; S6; T5; T5; D5; T5; S4; T5; D5; T5; D5; D5; D5; T5; D5; S4; D5; D5; T5; D5; T5; D5; T5; T3; S4; T5; S4; S6; T5; D5; T3; T3; D2; D5; T5; T3; S4; T5; S4; S4; T3; T5; T5; S6; D5; D4; T3;
T1; T1; S1; T1; T1; T1; S1; T1; T1; D2; T3; S4; T3; D2; T1; D5; D5; D7; T1; D2; S2; D5; D4; T3; D2; T1; T1; S1; T1; S4; S4; T3; D2; T1; T1; D5; D5; T1; T1; S2; T3; S4; S4; T1; T1; T3; S4; D5; D5; T1;
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Figure 1: A sample of the dataset, showing the hymn tune "Americus", by Charles H. Gabriel.

Results

Figure 2 is a graph showing the six scale degree classes in the hidden Markov model and their membership. The arrows show the likelihood of one of the hidden states proceeding to another. The six scale-degree categories are:

NCT Non-chord tones like to go to PSs (33% of the time), then Fs (22%), then ATs (20%).

CD Chordal dissonances like to go to ATs (43%).

AT Active tones like to go to Fs (34%) and other ATs (33%).

PS Primary stabilizers like to go to Fs (36%).

SS Secondary stabilizers like to go to Fs (27%) and ATs (25%).

F Foundations like to go to other Fs (54%).

This model shows some mild tendencies for scale-degree functions to move to certain others, but it cannot provide information about whether a given scale-degree function likes to move up or down, or by common tone, step, or skip. While the voice-leading information in this model is too general and too abstract to be useful in deriving normative voice-leading tendencies, it does tell us how to sort the scale degrees into categories based on similar voice leading behaviors. Table 1 replicates the distribution of scale degrees in Figure 2.

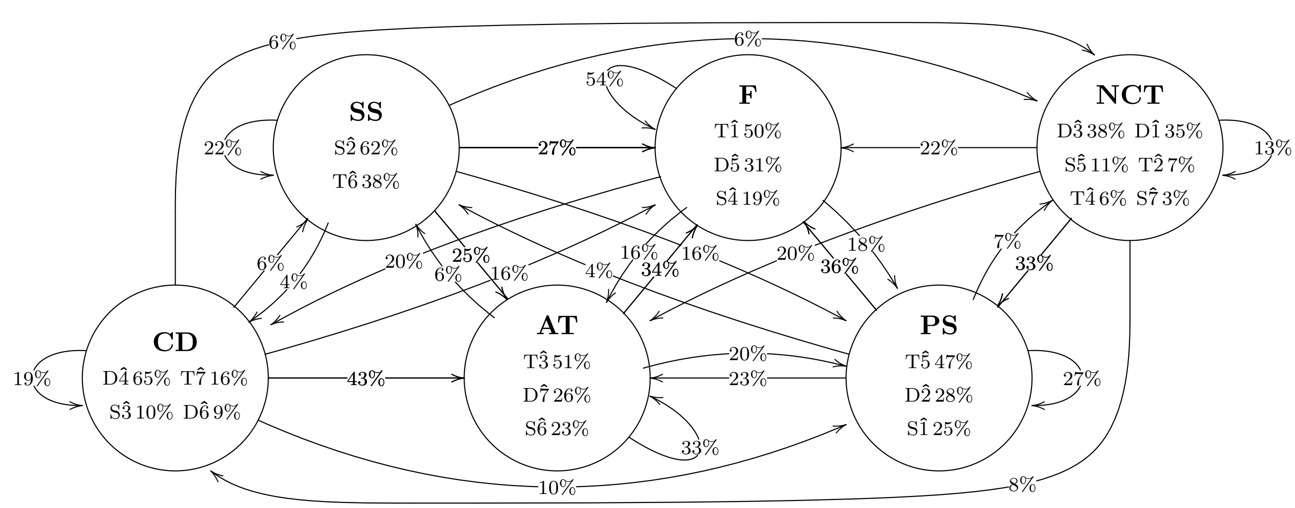


Figure 2: Hidden Markov model of scale-degree functions, with their membership and frequency within each circle and each function's most common paths expressed with arrows.

Table 1: A different representation of the scale-degree categories, arranged by scale degree number and harmonic function.

Scale Degree		Tonic	Subdominant	Dominant
$\hat{1}$	*Do*	Foundation	Primary Stabilizer	Non-Chord Tone
$\hat{2}$	*Re*	Non-Chord Tone	Secondary Stabilizer	Primary Stabilizer
$\hat{3}$	*Mi/Me*	Active Tone	Chordal Dissonance	Non-Chord Tone
$\hat{4}$	*Fa*	Non-Chord Tone	Foundation	Chordal Dissonance
$\hat{5}$	*Sol*	Primary Stabilizer	Non-Chord Tone	Foundation
$\hat{6}$	*La/Le*	Secondary Stabilizer	Active Tone	Chordal Dissonance
$\hat{7}$	*Ti/Te*	Chordal Dissonance	Non-Chord Tone	Active Tone

In order to distinguish how the scale-degree functions might tend to resolve differently in each harmonic context, I then created a Markov model with 21 states, one for each of the row/column combinations in Table 1, to see how each behaves. Because there are 21 choose 2, or 210, possible voice-leading connections, I have divided it into three charts, found in Figure 3. To declutter the graph, I have also removed some connections that occur less than 5% of the time. Figure 3a shows the most common resolutions in authentic progressions (T $\rightarrow$ S, S $\rightarrow$ D, or D $\rightarrow$ T). The common patterns for each scale-degree function can be summarized as:

- NCTs (T$\hat{2}$ T$\hat{4}$ S$\hat{5}$ S$\hat{7}$ D$\hat{1}$ D$\hat{3}$) move by step or hold as a common tone.
  - The exception is D$\hat{3}$, which usually skips down to T$\hat{1}$.
- CDs (T$\hat{7}$ S$\hat{3}$ D$\hat{4}$ D$\hat{6}$) resolve down by step (73%) or hold into a suspension.
- ATs (T$\hat{3}$ S$\hat{6}$ D$\hat{7}$) discharge as described in Harrison 1994 (Table 2)
  - If they do resolve irregularly, it is by step or third in the opposite direction.
  - The weakest of these tendencies is S$\hat{6}$ to D$\hat{5}$, which moves upward to D$\hat{7}$ 34% of the time.
- PSs (T$\hat{5}$ S$\hat{1}$ D$\hat{2}$) resolve by step.
  - S$\hat{1}$ also commonly holds into a suspension.
- SSs (T$\hat{6}$ S$\hat{2}$) hold as common tones or descend by third or fifth.
- Fs (T$\hat{1}$ S$\hat{4}$ D$\hat{5}$) most often progress to the next harmonic function's foundation.
  - Secondary resolutions are by common tone, step, or third (for when they are doubled).

Figure 3b shows the most common voice-leading connections in plagal progressions (T $\rightarrow$ D, D $\rightarrow$ S, or S $\rightarrow$ T), and the trends for each scale degree function are:

- NCTs (T$\hat{2}$ T$\hat{4}$ S$\hat{5}$ S$\hat{7}$ D$\hat{1}$ D$\hat{3}$) move by step or hold as a common tone
  - They also have secondary leaping tendencies: T$\hat{2}$ $\rightarrow$ D$\hat{5}$, S$\hat{7}$ $\rightarrow$ T$\hat{5}$,and S$\hat{5}$ $\rightarrow$ T$\hat{1}$.
- CDs (T$\hat{7}$ S$\hat{3}$ D$\hat{4}$ D$\hat{6}$) hold as a common tone.
- ATs (T$\hat{3}$ S$\hat{6}$ D$\hat{7}$) are less likely to follow Harrison 1994 than in authentic progressions.
  - When they resolve irregularly, it is usually by step or third in the opposite direction.
- PSs (T$\hat{5}$ S$\hat{1}$ D$\hat{2}$) hold as a common tone.
- SSs (T$\hat{6}$ S$\hat{2}$) resolve to the same pitch that the PS would.
- Fs (T$\hat{1}$ S$\hat{4}$ D$\hat{5}$) move much more often by step than in authentic progression.

As Figure 3c suggests, when no function change occurs, motion among foundations, active tones, primary stabilizers, secondary stabilizers, and chordal dissonances (the functional scale degrees) is unrestricted, while non-chord tones have stepwise resolution tendencies to a more stable adjacent scale degree.

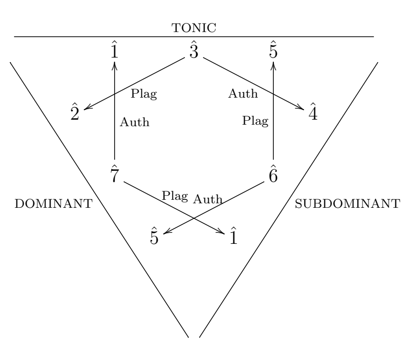
While the data is noisy, and the resolution tendencies are numerous and complicated, some scale-degree functions have distinct tendencies with 80% or more of their uses in the corpus following only one or two paths. These scale-degree functions with clear stylistic norms are the non-chord tones, chordal dissonances, and active tones, while the foundations and stabilizers have more possibilities for unusual resolutions without disturbing the musical style. These results suggest that emulating these patterns will result in more stylistic part writing with fewer errors.

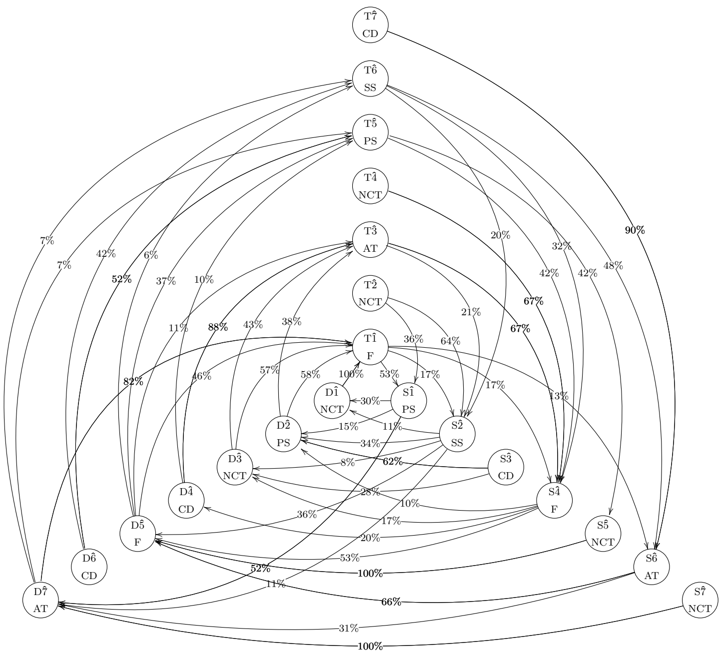
Table 2: Active Tone Resolution Tendencies

(a) Harrison's Figures 3.2–3.3: Agent Discharges

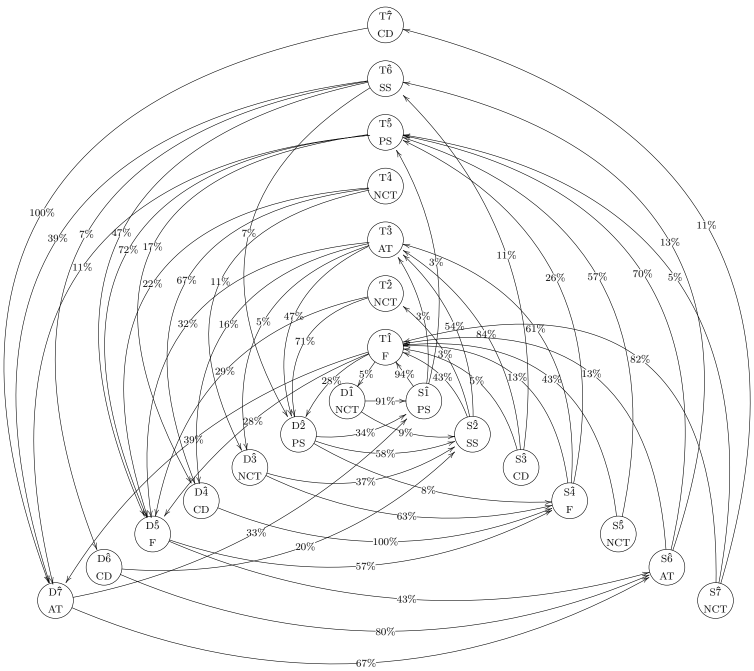
$\hat{3}$	$\hat{4}$	T – S	discharge
$\hat{3}$	$\hat{2}$	T – D	discharge
$\hat{6}$	$\hat{5}$	S – T, S – D	discharge
$\hat{7}$	$\hat{1}$	D – S, D – T	discharge

(b) Chart of Active Tone Resolutions

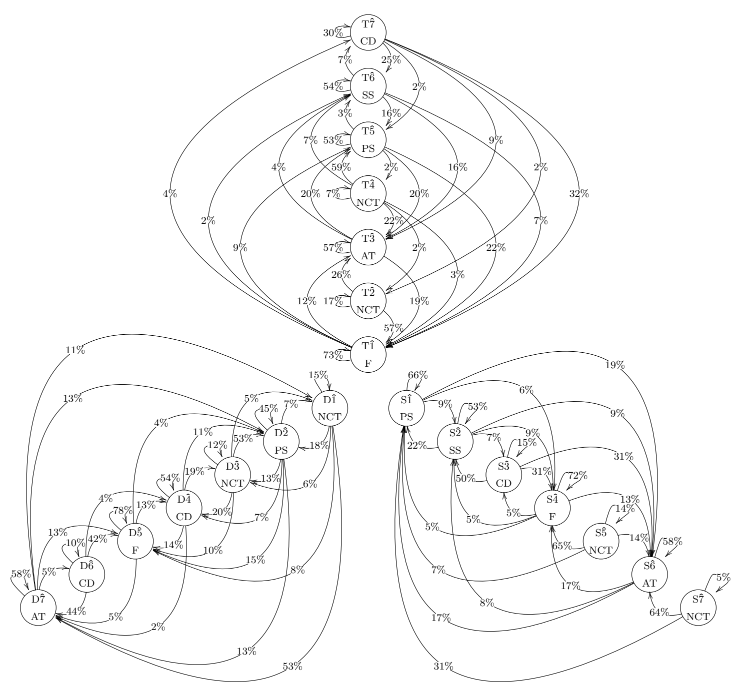




(a) Authentic Progression Scale-Degree Tendencies



(b) Plagal Progression Scale-Degree Tendencies



(c) Scale-Degree Tendencies with No Change of Harmonic Function

Figure 3: Resolution Tendencies of All Scale-Degree Functions in All Harmonic Contexts

Discussion

The voice-leading procedure chart in Table 3 distills the scale-degree tendency data into a set of best practices for note-to-note part writing. Because the chart is somewhat oversimplified, in the classroom I would spend time fleshing these procedures out. For example, I introduce students to all the different types of non-chord tones labeled as "others" in Table 3. I also devote significant class time to showing students how the downward stepwise resolution of chordal dissonances can be suspended by a non-chord tone, especially in the cadential six-four. As students learn to follow these tendencies consistently, I also begin to relax my requirements for resolving active tones. Specifically, improper active tone resoluations are permissible in inner voices, in keyboard style, and in certain types of progressions where S$\hat{6}$ ascends to D$\hat{7}$ or T$\hat{7}$. Students also need to know that modal mixture is a special type of chromaticism that does not create true inflected foundations or stabilizers.

This chart is listed in order of priority, with the most important scale-degree functions to resolve according to the model at the top. The first four scale-degree functions (NCT, CD, AT, and Inflected F, PS, or SS) are considered tendency tones, while the bottom three (PS, SS, and F) are better treated in practice by attending to avoiding illegal consecutive perfect fifths and octaves and following the law of the shortest way.

Table 3: Voice-Leading Procedures

	No Function Change $T \to T, S \to S, D \to D$	Authentic Progression (AP) $T \to S, S \to D, D \to T$	Plagal Progression (PP) $T \to D, D \to S, S \to T$
NCT	double neighbor = 3rd, others = step	anticipation = common tone, escape tone = down by 3rd, others = step	
CD	free	down by step	common tone
AT	free	step: Mi/Me → Fa, Mi/Me → Re, La/Le → Sol, Ti/Te → Do	
Infl. F, PS, or SS	common tone, step, or diminished 3rd	step or diminished 3rd in the direction of its inflection	
PS	free	step	common tone
SS	free	common tone, down by 3rd or 5th, or up by 4th	step
F	free	free	free

I have constructed an entire set of simple part-writing procedures around the core voice-leading principles outlined in Table 3. Before learning about voice leading, students must first learn the best practices involved in writing good four-part chorale-style music that I have outlined in Table 4. I also introduce my students to the phrase model, and the prevalence of authentic progressions over plagal progressions. After careful practice with harmonic analysis, students should be able to explain any plagal progressions between two chords as part of a prolongational progression, a sequential progression, or certain types of cadential progression. By placing their chord symbols for embellishing chords and sequential progressions in parentheses, students can practice reducing any phrase of music down to the basic phrase model.

In order to complete a part-writing exercise, students also require some guidance on doubling. The doubling procedures in Table 5 also derive from a statistical analysis, using a slightly different corpus: Instead of hand selecting four-voice chorales to form the dataset, I computationally selected four-voice triads from the entire set of Cyber Hymnal MIDI files. This allowed for a larger sample size of 63,496 triads in four parts, instead of approximately 15,000 chords in the original dataset.

The Kostka-Payne and Burstein-Straus undergraduate theory textbooks both advocate doubling the bass in root-position and second-inversion triads, and doubling the most convenient note of first-inversion triads. Figure 4a shows that the practice of hymn composers clearly supports this doubling model, with the bass doubled in 91% of both root-position and first-inversion triads. For first inversion chords, however, these doubling procedures give students no advice on how to decide what to double.

Figure 4b shows that the corpus also clearly supports the scale-degree- function-based doubling procedures that I devised. Furthermore, these doubling procedures always give a recommendation of what tone to double, and never leave the choice up to the student's inexperienced judgment.

Figure 4c compares the two models side by side. The standard inversion-based doubling model gives specific doubling advice that is corroborated by the corpus 81.5% of the time, while the scale-degree-function-based model gives advice that is corroborated by the corpus 88.7% of the time. In chord-tone doubling, as in voice leading, this approach has the pedagogical advantage of offering students more guidance in making decisions in all part-writing situations.

The voice-leading procedures nevertheless sometimes give conflicting advice. For example, resolving tendency tones and avoiding illegal consecutive perfect unisons, fifths, and octaves may sometimes require an incomplete chord. Students need to know that it is preferable to have an incomplete chord than to have bad tendency tone resolutions or parallels. The chart in Table 6 combines the choral writing procedures, doubling procedures, and voice-leading procedures into a comprehensive priority list. Students can also use this chart as a part-writing checklist.

Table 4: Four-part chorale-style vocal writing guidelines

Vocal Ranges — *Stay within the following range for each part:*

S	=	C4	–	G5
A	=	F3	–	C5
T	=	C3	–	G4
B	=	F2	–	C4

Spacing — *Do not exceed the following distances between adjacent parts:*

Maximum distance between S and A	=	1 octave
Maximum distance between A and T	=	1 octave
Maximum distance between T and B	=	unrestricted

Voice Crossing and Overlap — *Keep the parts from encroaching on each other's ranges by avoiding the following situations:*

- **Voice Crossing**
  - A voice sounds below the next lower voice part
  - A voice sounds above the next higher voice part
- **Voice Overlap**
  - A moves below the previous note in the next lower part
  - A moves above previous note in the next higher part

Voice Independence — *The texture should always sound like it has four distinct voices moving independently:*

- **Illegal Consecutive Intervals** *occur when:*
  - Voices a perfect unison, fifth, or octave apart (or their compound)
  - Step or skip to the same interval (or octave equivalent)

Chord Completion — *Triads should contain all three notes, and sevenths all four notes, with the following exception:*

- Voice leading sometimes requires the primary stabilizer to be left out of one or more chords.
  - When root position V^7 resolves to root position I, one of the chords must omit its primary stabilizer.

Law of the Shortest Way — *It is easier to sing more conjunct lines:*

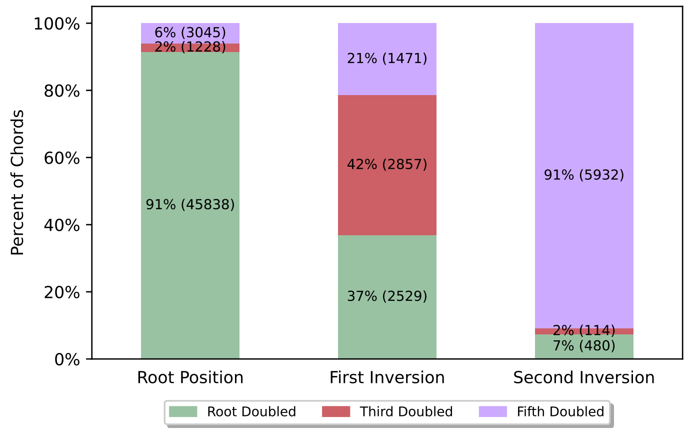
- Avoid leaps by sixths, sevenths, and chromatic intervals.
- Prefer the closest or second closest chord tone in the next chord.

Table 5: Doubling Procedures

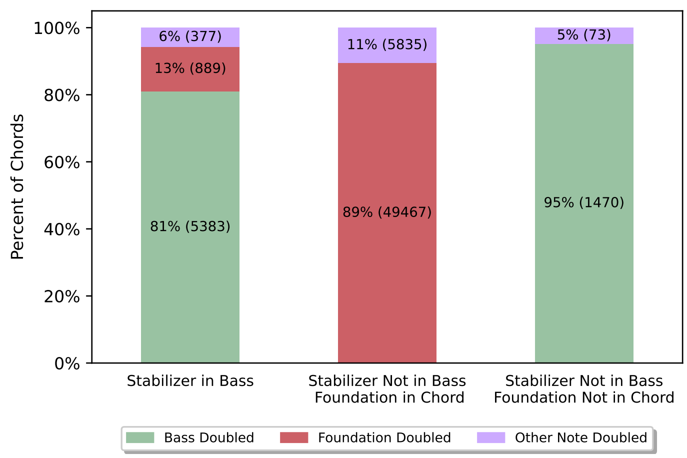
1. If a primary or secondary stabilizer is in the bass voice, double it.
2. If not, then if the foundation is in the chord, double it.
3. If not, then double the bass note.
4. Voice leading may overrule the doubling indicated by the first three rules.
5. Never double dominant active tones, chordal dissonances, or inflections other than modal mixture.

Table 6: Part-Writing Priorities

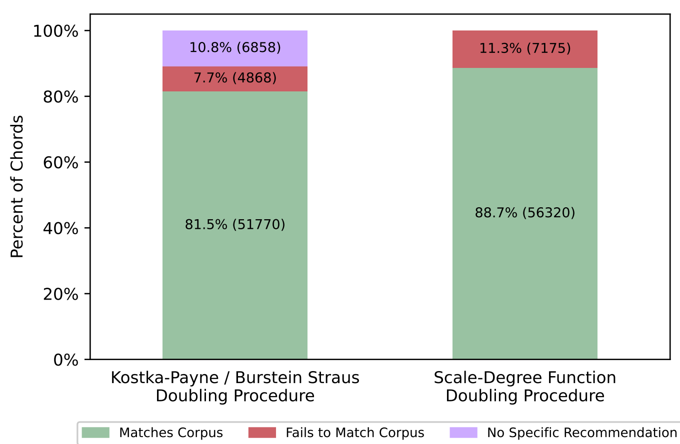
1. Top priorities	2. Important considerations	3. Do whenever possible
tendency tone resolutions	law of the shortest way	AT resolutions in inner voices
doubling prohibitions	avoiding consecutive P5s	doubling procedures 1–3
avoiding consecutive P1s & P8s	including the 5th in triads	including the 5th in 7th chords
range, spacing, avoiding crossing	avoiding voice overlaps	CD resolutions in passing chords



(a) The corpus supports Kostka-Payne's and Burstein-Straus's doubling procedure. As they recommend, the bass is doubled in 91% of root position and second inversion triads, and there is no clear doubling preference for first inversion chords.



(b) The corpus also supports the scale-degree function doubling procedures. 81% of chords with a stabilizer in the bass have their bass note doubled, while more than 90% of remaining chords have the foundation doubled when present and the bass doubled when not.



(c) The scale-degree function doubling procedure gives a specific doubling matching the corpus more consistently than the Kostka-Payne/Burstein-Straus procedure.

Figure 4: Comparison of chord-inversion-based and scale-degree-function-based doubling procedures. The scale-degree-function doubling procedure provides students more guidance in deciding which chord tone to double.

Figure 5 provides the score for the hymn tune "Americus", which was used in Figure 1 to demonstrate data format used to create the hidden Markov model. There is a modulation in the hymn's first line, but the fact that the computer analyzed the scale degrees entirely in the key of E did not create serious issues for the hidden Markov model. identifying this passage's scale-degree functions in the key of E instead of B will only cause the D♯ soprano note of the half-note cadential six-four to seem to resolve irregularly. Modulation to less closely related keys will, of course, cause more noise in the corpus data than this example does, but church hymns mostly stay pretty close to home.

The analysis of embellishing chords benefits from the use of two analysis layers: the chord's local harmonic function based on its scale-degree content, and a reinterpretation of the chord as part of a multi-chord harmonic functional zone, shown in brackets. There are also two sets of scale-degree functions that correspond to these two harmonic functions. I advise my students to try to resolve all tendency tones properly, and when conflicts arise between the two layers, prefer the tendency tones listed closer to the top of the voice-leading procedures chart.

There are many examples of chromatic chords in the dataset, like the applied dominants on the second line of Figure 5. With our universal voice-leading recipe, a theory teacher no longer has to spend time specifically teaching the resolutions of the $\flat\hat{2}$ in Neapolitans, the bass and $\sharp\hat{4}$ in Augmented sixth chords, or the inflected pitches in common-tone diminished sevenths. This allows for much faster coverage of each type of chromatic harmony in the undergraduate theory curriculum.

The chromatic inflections of modal mixture (e.g. in m. 11 of Figure 5) do not affect the tendencies of the scale-degree functions. The same is true of applied dominants (e.g. mm. 7 and 10 of Figure 5), although I also use a two-layer analysis system to teach these to my students, where the bracketed chord symbol ignores the secondary key as do the scale-degree functions. The resolution of extended tertian chords is also already baked into the scale-degree function model.

The model only begins to break down when approaching the extreme chromaticism of some late-19th- and early-20th-century composers. Having a grounding in this way of approaching analysis, however, provides a much easier segue into interpreting this music from a dualist harmonic perspective, as in Harrison 1994.

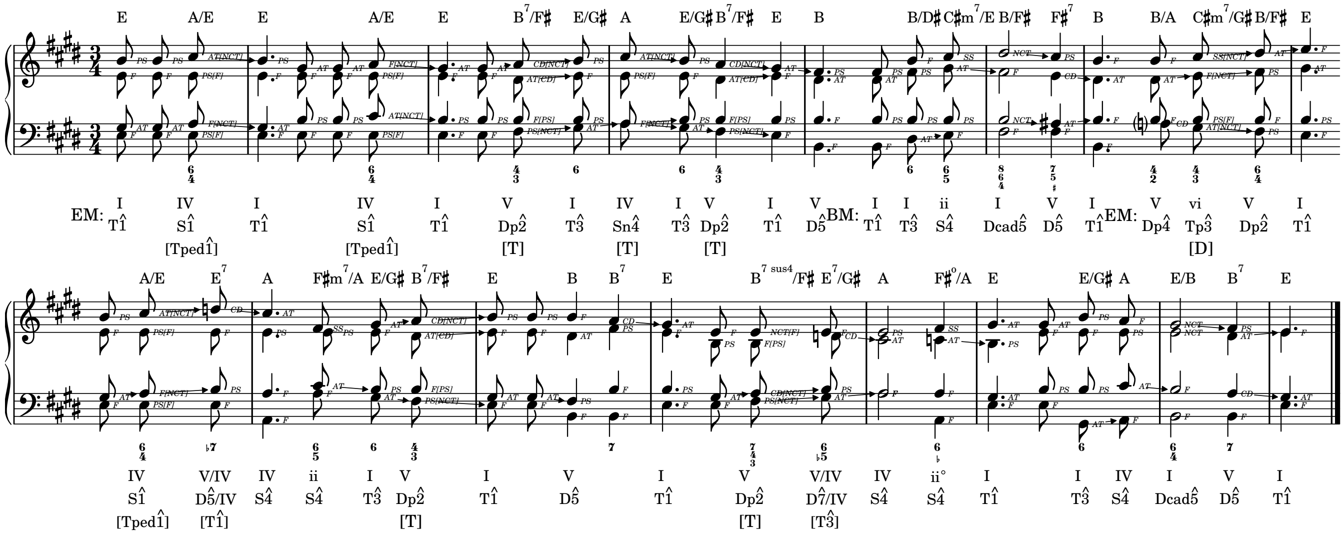


Figure 5: Analysis of "Americus", by Charles H. Gabriel, showing functional-bass symbols, scale degree function abbreviations for each note, and resolution arrows for all tendency tones.

Conclusions

This study has revealed stylistic voice-leading trends in tonal music with considerable explanatory power. Not only can they shape the analysis of tonal music as a dialogue with these stylistic norms, they also offer undergraduate music majors precise tools for emulating tonal harmonic practice. With the voice-leading principles outlined in Table 3, students will always have guidelines for resolving each note in a chord. The list of principles and procedures is not significantly shorter than in traditional part-writing methodology, but its application is far more comprehensive. With the same care that undergraduate theory teachers apply to traditional tonal harmony pedagogy, this system has the potential to significantly improve students' tonal analysis and part writing skills.

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